Content and composition

This package offers a sound introduction to the mathematics that we use to study the real world around us. Partial differential equations are central concepts in the description of many real-world phenomena. Measure theory and Integration theory provide the tools with which we can study these equations and many other concepts. Finally, Tensor calculus and Differential geometry are the methods with which we study properties of curved space and curved surfaces, which appear in a wide variety of applications.

This package is flexible enough not only to open the mathematics students access to modern mathematics (helping them choose a study/research direction for later), but it also provides the student in physics, chemistry or engineering the theoretical tools needed for understanding the fundamentals of quantum mechanics, statistical mechanics, continuum mechanics, theory of relativity, and medical imaging, for instance.

A note for non-mathematics students: these courses are aimed at students with a mathematical background, but can be followed by any student who is willing to brush up on the mathematics needed. As an indication of what provides a minimal mathematical background, you should be familiar with the concepts of theorem and proof, and with some basic notions of linear algebra (vectors, matrices, linear operators, eigenvalues and eigenvectors, diagonalization of matrices) and of calculus (differentiation, partial derivatives, Gauss theorem, integration by parts). For Partial differential equations you also need to be familiar with ordinary differential equations. For Measure theory and Integration theory and for Tensor calculus and Differential geometry some basic elements of topology (open and closed sets, compactness, continuity) and of functional analysis (normed spaces, convergence in normed spaces) are needed.

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Course description

Partial differential equations
For reliable answers to important questions such as:
- How is the weather going to be tomorrow or next week?
- What is the fate of the CO2 stored in the subsurface?
- How do nutrients reach (human) tissues?
- Has the coachwork in a new auto-design sufficient stiffness?

one needs to translate the processes into mathematical models. A large part of such models are partial differential equations. This course presents the basic principles and techniques for understanding and analyzing such equations.

Students of non-mathematics majors are advised to follow this course in their third year.

Measure, integration, and probability theory
Measuring is a elementary activity of daily life. The measure of a physical quantity is often derived from other measured values such as volume, mass, charge, and probability. The mathematical description of the measuring activity is contained in the theory of measure and integration, a basic topic of modern mathematics. Students in mathematics and related fields need to know its fundamentals. This course offers a basic introduction to the mathematical construct of measure and integral, Lebesgue’s theory, with a motivation from probability theory and modern analysis.

Tensor calculus & differential geometry
A tensor is a multilinear extension of a linear map. Tensors are encountered in virtually any context in which linearization, a powerful generic technique used in mathematical modeling, plays a key role.Examples include continuum mechanics, electromagnetism, thermodynamics, relativity theory, and image analysis. The study of tensors is referred to as tensor calculus.

In many cases one considers tensor fields, i.e. tensor-valued functions defined on some base manifold, in practice usually space or spacetime. One then enters into the realm of differential geometry, which simultaneously studies both the pointwise and neighbourhood interactions between the various tensorial quantities defined on this manifold.

The course Tensor calculus & differential geometry deemphasizes physical interpretations, but focuses instead on the generic mathematical machinery. Practical examples will illustrate the beauty of the theory.